LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2009

ST 1815 / 1810 - ADVANCED DISTRIBUTION THEORY

Date & Time: 09/11/2009 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION – A Answer all the questions (10 x 2 = 20)

01. Define truncated distribution and give an example.
02. Examine whether truncated Poisson distribution, truncated at zero, is a power series distribution
03. Define Lognormal distribution and find its median.
04. If X is Inverse Gaussian, show that 2X is also Inverse Gaussian.
05. Define a bivariate Poisson distribution.
06. Let (X_1, X_2) have a bivariate binomial distribution. Find the marginal distributions.
07. Show that the exponential distribution satisfies lack of memory property..
08. Let (X_1, X_2) have a bivariate normal distribution. Show that $X_1 + X_2$ has a normal distribution.
09. Let X_1, X_2, X_3 be independent standard normal variables. Examine whether $2X_1^2 + 5X_2^2 + 3X_3^2 - 2X_1X_2 + X_1X_3$ is distributed as chi-square.
10. Let X be $B(2, \theta)$, $\theta = 0.1, 0.2$. If θ is discrete uniform, find the mean of the compound distribution.

SECTION – B Answer any five questions (5 x 8 = 40)

11. State and establish a characterization of exponential distribution based on order statistics.
12. State and establish the additive property satisfied by bivariate binomial distribution.
13. Find the regression equations associated with Bivariate Poisson distribution.
14. Derive the mean and the variance of Inverse Gaussian distribution.
15. Show that the Geometric mean of independent Lognormal variables is Lognormal.
16. Define Bivariate exponential distribution and show that the marginals are exponential.
17. Find the mean and the variance of non-central t - distribution.
18. Given a random sample from a normal distribution, show that the sample mean and the sample variance are independent, using the theory of quadratic forms.

SECTION – C Answer any two questions (2 x 20 = 40)

19 a) State and establish the characterization of geometric distribution based on conditional distribution.

b) Derive the cumulant generating function for a power-series distribution. Hence obtain a recurrence relation satistfied by the cumulants.

20 a)Let X_1 and X_2 be two independent normal variables with the same variance. State and establish a necessary and sufficient condition for two linear combinations of X_1 and X_2 to be independent.

b) State and establish the additive property of bivariate normal distribution.

21a) Let (X_1, X_2) have equal marginal absolutely continuous bivariate exponential distribution(Block and Basu). Show that the marginal distributions are not exponential.

b) Define non-central chi-square variable and derive its distribution.

22 a) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed normal variables with mean zero and the variance σ^2. Show that $\mathbf{X}^{/} A\mathbf{X} / \sigma^2$ is distributed as chi-square if A is idempotent.

b) Let (X_1, X_2) have trinomial distribution with index n and cell probabilities θ_1, θ_2. Assuming that (θ_1, θ_2) has a uniform distribution, find the compound distribution of (X_1, X_2).

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